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Business Cycle Dependent Unemployment Insurance

by Torben M. Andersen and Michael Svarer

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Business Cycle Dependent Unemployment Insurance*

FIRST AND PRELIMINARY DRAFT

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Abstract

The consequences of cylical contingencies in unemployment insurance systems are considered in a search-matching model allowing for shifts between "good" and "bad" states of nature. An argument for state contingencies is that insurance arguments are stronger and incentive effects weaker in "bad" than in "good" states of nature. We confirm this and show that cyclically dependent benefit levels not only provide better insurance but may have structural effects implying that the structural (average) unemployment rate decreases, although the variability of unemployment may increase.

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1 Introduction

Optimal unemployment insurance systems trade-off incentives and insurance. Since unemployment risk is intimately related to the business cycle situation it is to be expected that the value of insurance is business cycle dependent. At the same time it may be conjectured that the distortions from unemployment insurance may be larger in periods with low unemployment, and vice versa. Both of these effects go in the direction of making optimal benefit levels counter-cyclical, that is, benefit generosity is high when unemployment is high, and low when unemployment is low.

Some countries have explicit rules linking elements of the unemployment insurance system to the state of the labour market. Probably the most sophisticated scheme is found in Canada where benefit eligibility, levels, and duration depend on the level of unemployment according to pre-determined rules [1]. The US has a system of extended benefit duration in high unemployment periods (see Committee on Way and Means (2004)). Other countries have pursued a more discretionary approach - in some cases in a semi automatic fashion[2] - by adjusting labour market policies to the state of labour market i.e. extending benefits or labour market policies in general in high unemployment periods and tightening the schemes in periods with low unemployment.

The is a large literature on the design of unemployment insurance schemes. Since Baily (1978) it is well-known that the optimal benefit level trades-off insurance and incentives. Recent work has extended these insights in various directions (for a survey see e.g. Frederiksson and Holmlund (2006)). Surprisingly there is neither a large theoretical literature on the effects of state dependent unemployment insurance, nor an empirical literature[3] exploring the state dependencies in the effects of various labour market policies including the benefit level. Kiley (2003) and Sanchez (2008) argue within a search framework that the initial benefit level should be higher and its negative duration dependence weaker in a business cycle downturn compared to an upturn. Both models are partial and rely on the assumption that benefits are more distortionary in a boom. In Andersen and Svarer

[1] See http://www.hrsdc.gc.ca/eng/ei/menu/eihome.shtml.

[2] Sweden is an example of a country which has used labour market policies in this way.

[3] The few exceptions are: Moffitt (1985), Arulampalam and Stewart (1995), Jurajda and Tannery (2003), and Røed and Zhang (2005). The first three of these studies finds that benefits affects incentives less in a downturn, whereas the study by Røed & Zhang does not find any differences in the effect of benefit on incentives across the business cycle.

(2008) it is shown that the optimality of countercyclical benefit levels depends not only on the possibility of using the public budget as a buffer but also whether distortions move pro-cyclically. In this case countercyclical unemployment benefits may also contribute to lower the structural (average) unemployment rate. However, the model is static and does not allow for changes in the business cycle situation.

This paper develops a search model in which the business cycle situation may change between "good" and "bad" times[4]. Matching frictions imply a co-existence of unemployed persons and vacant jobs, but the underlying job separation rates and job finding rates are business cycle dependent. The unemployment benefit scheme is tax financed and benefits are allowed to be state contingent. Since the main issue in this paper is the trade-off between insurance and incentive, the model is cast in such a way that it focusses on how unemployment benefits affect job search incentives. The key task is to work out the implications of state dependent benefit levels on the unemployment rate, and the welfare implications of such a dependency. State dependent unemployment benefits strengthen also automatic stabilizers which may have effects via aggregate demand effects. Such effect do not arise in the present framework which is has focus on the structural consequences of state dependent benefit levels.

It is shown that the possible change in the business cycle situation has an important effect on search behaviour and therefore unemployment and other key variables. The reason is that agents perceive the possibility of a change in the business cycle situation and this affects the search behaviour of unemployed. Clearly this effect depends on both the difference between the two states and the likelihood of a change in the business cycle situation. It is an implication that the incentive effects of a given benefit level or changes herein differ between upturns and downturns, with distortions of search behaviour being largest in upturns. Allowing benefits to depend on the business cycle situation may therefore have important effects on search behaviour and the unemployment rate (both in the different states of nature and on average across the states of nature). It is among other things shown that higher benefits in a downturn and lower benefits in an upturn may increase search effort in both

[4] The main modeling difficulty here is to ensure stationarity of public finances under a tax financed unemployment insurance scheme. This is ensured by the specific assumptions concerning state transitions and the tax policy.

states of nature, and therefore cause a fall in unemployment in both states. This arises if the business cycle situation is not very persistent and agents in a downturn perceive a high probability of a shift to an upturn with a higher job finding rate. Optimal state contingent benefits have higher benefits in downturns than upturns, and this is shown to lower structural (average) unemployment, but it may be achieved at the cost of more variability in the unemployment rate (higher in downturns, and lower in upturns). This shows that state contingent benefits levels may improve insurance without jeopardizing structural concerns.

The paper is organized as follows: The model is set-up in section 2, and as a prelude to the subsequent discussion section 3 briefly considers as a benchmark case the one state version of the model. The main results are given in section 4 exploring both the consequences of state dependent benefit levels and the optimal benefit levels. Concluding remarks are given in section 5.

2 A search matching model with business cycles

Consider a standard search matching model of the Pissarides-Mortensen type in continuous time (see e.g. Mortensen and Pissarides (1994) and Pissarides (2000)). All workers are ex-ante identical and have the same productivity in work. Workers search for jobs but a matching friction implies that unemployment and vacancies coexist. Firms create vacancies, and filled jobs are destructed by some exogenous separation rate p ($p \in [0,1]$). All probabilities are parameters of the associated time homogeneous Poisson process.

The state of nature evolves between two states *good* (G) and *bad* (B) with the following (symmetric) transition[5] probabilities[6]

present\past state	B	G
B	π	$1-\pi$
G	$1-\pi$	π

where $0 \leq \pi \leq 1$. This formulation captures that if the economy is in a boom (recession) this state of nature may continue with probability π and terminate and turn into a recession (boom) with probability $1-\pi$. Hence, π is also a measure of the persistence in the current business cycle situation.

The job separation rate p is in the four possible states of nature given as follows

present\past state	B	G
B	p_{BB}	p_{BG}
G	p_{GB}	$p_{GG} < p_{BB}$

i.e. the basic transition is between a regime with either a low level (p_{GG}) or high level ($p_{BB} > p_{GG}$) of job separations. Upon transition there is an extraordinary high ($p_{BG} > p_{BB}$) or low ($p_{GB} < p_{GG}$) level of job separations (see below).

There is an unemployment benefit scheme providing a flow benefit b to unemployed workers and it is financed by a proportional wage income tax (τ) and a lump sum tax (T) (see below). The inclusion of lump sum taxes is done to make the analysis involving four possible states of nature and public budget effects more transparent. The key problem is that the budget balance in general will display path dependence, to cope with this and to ensure stable debt levels, policies will in general

[5] We assume a symmetric transition matrix to simplify the analysis. Empirical evidence indicates some asymmetry with more persistence in good than in bad business cycle situations. The estimated value of π in discrete models on quaterly data is in the range 0.7 to 0.9, see Hamilton (1994). In a three state model (recession, normal and high growth) somewhat higher levels of persistence are found, see Artis et al. (2004).

[6] Note that the unconditional stationary probability of being in a given state B or G is

$$\Pr(G) = \Pr(B) = \frac{1}{2}$$

The unconditional probabilities of the four possible states are: $\Pr(BB) = \Pr(GG) = \frac{1}{2}\pi$ and $\Pr(GB) = \Pr(BG) = \frac{1}{2}(1-\pi)$.

have to be path dependent. This is captured via the lump-sum tax, while the tax rate is assumed constant and the benefit level may depend on whether the state is "good" or "bad". Note that there are no marginal labour supply decisions (intensive margin) in the following so the use of lump sum taxation does not affect any results, but serves the purpose of making the exposition more simple and transparent.

2.1 Individual utility and search effort

Consider an infinite number of identical households, and normalize the population size to unity. Employed workers receive a wage w and work l hours which both are state independent and hence the instantaneous utility can be written

$$h\left(w\left[1-\tau\right]-T_{ij}\right)+e(1-l)$$

where τ is the state independent income tax rate and T_{ij} is the lump sum tax paid if the current state is i and the previous state j. Working hours l are exogenous and the time endowment has been normalized to 1. Both $h()$ and $e()$ are concave functions. The instantaneous utility for unemployed is

$$g\left(b_i-T_{ij}\right)+f(1-s_{ij})$$

where s_{ij} is time spent searching for a job if the current state is i and the previous state j. The utility functions g and f are concave, and the assumption that the utility of income for unemployed differs from that of employed makes it possible to capture possible stigmatizing effects of unemployment (if $g(y)\leq h(y)$ for all y and/or $f(1-x)\leq e(1-x)$ for all x). Note that the separability assumption ensures that search is not dependent on current income (see below)[7]. In addition, note that the benefit level only takes two values conditional on the current state, whereas the lump sum tax also depends on the past state. This results in four different levels of net compensation to the unemployed.

[7] There is no on-the job search since all jobs are assumed identical and have the same wage.

Value functions

Consider first the value functions for currently employed workers (W_{ij}^{E}) in give current state (i) and past state (j). Note that eventual job separation affects the future labour market position. We have

$$\rho W_{BB}^{E} = h(w\left[1-\tau\right] - T_{BB}) + e(1-l) + \pi p_{BB}\left[W_{BB}^{U} - W_{BB}^{E}\right]$$
$$+ (1-\pi)\left[(1-p_{GB})\left[W_{GB}^{E} - W_{BB}^{E}\right] + p_{GB}\left[W_{GB}^{U} - W_{BB}^{E}\right]\right]$$
$$\rho W_{BG}^{E} = h(w\left[1-\tau\right] - T_{BG}) + e(1-l) + W_{BB}^{E} - W_{BG}^{E} + \pi p_{BB}\left[W_{BB}^{U} - W_{BB}^{E}\right]$$
$$+ (1-\pi)\left[(1-p_{GB})\left[W_{GB}^{E} - W_{BB}^{E}\right] + p_{GB}\left[W_{GB}^{U} - W_{BB}^{E}\right]\right]$$
$$\rho W_{GG}^{E} = h(w\left[1-\tau\right] - T_{GG}) + e(1-l) + \pi p_{GG}\left[W_{GG}^{U} - W_{GG}^{E}\right]$$
$$+ (1-\pi)\left[(1-p_{BG})\left[W_{BG}^{E} - W_{GG}^{E}\right] + p_{BG}\left[W_{BG}^{U} - W_{GG}^{E}\right]\right]$$
$$\rho W_{GB}^{E} = h(w\left[1-\tau\right] - T_{GB}) + e(1-l) + W_{GG}^{E} - W_{GB}^{E} + \pi p_{GG}\left[W_{GG}^{U} - W_{GG}^{E}\right]$$
$$+ (1-\pi)\left[(1-p_{BG})\left[W_{BG}^{E} - W_{GG}^{E}\right] + p_{BG}\left[W_{BG}^{U} - W_{GG}^{E}\right]\right]$$

where ρ is the relevant exogenous interest rate. The value function for current unemployed workers in a given current state (i) and a past state (j) is denoted W_{ij}^{U}. Note that job search influences the future labour market position. We have

$$\rho W_{BB}^{U} = g(b_B - T_{BB}) + f(1-s_{BB}) + \pi\alpha_B s_{BB}\left[W_{BB}^{E} - W_{BB}^{U}\right]$$
$$+ (1-\pi)\left[(1-\alpha_G s_{BB})\left[W_{GB}^{U} - W_{BB}^{U}\right] + \alpha_G s_{BB}\left[W_{GB}^{E} - W_{BB}^{U}\right]\right]$$
$$\rho W_{BG}^{U} = g(b_B - T_{BG}) + f(1-s_{BG}) + W_{BB}^{U} - W_{BG}^{U} + \pi\alpha_B s_{BG}\left[W_{BB}^{E} - W_{BB}^{U}\right]$$
$$+ (1-\pi)\left[(1-\alpha_G s_{BG})\left[W_{GB}^{U} - W_{BB}^{U}\right] + \alpha_G s_{BG}\left[W_{GB}^{E} - W_{BB}^{U}\right]\right]$$
$$\rho W_{GG}^{U} = g(b_G - T_{GG}) + f(1-s_{GG}) + \pi\alpha_G s_{GG}\left[W_{GG}^{E} - W_{GG}^{U}\right]$$
$$+ (1-\pi)\left[(1-\alpha_B s_{GG})\left[W_{BG}^{U} - W_{GG}^{U}\right] + \alpha_B s_{GG}\left[W_{BG}^{E} - W_{GG}^{U}\right]\right]$$
$$\rho W_{GB}^{U} = g(b_G - T_{GB}) + f(1-s_{GB}) + W_{GG}^{U} - W_{GB}^{U} + \pi\alpha_G s_{GB}\left[W_{GG}^{E} - W_{GG}^{U}\right]$$
$$+ (1-\pi)\left[(1-\alpha_B s_{GB})\left[W_{BG}^{U} - W_{GG}^{U}\right] + \alpha_B s_{GB}\left[W_{BG}^{E} - W_{GG}^{U}\right]\right]$$

In the special case where $h\left(w\left[1-\tau\right] - T_{ij}\right) = w\left[1-\tau\right] - T_{ij}$ and $g(b_i - T_{ij}) = b_i - T_{ij}$ the value function for both employed and unemployed is giving the expected present value of income (net of disutility from work/search). This case can therefore be interpreted as reflecting a situation with a perfect capital market allowing individuals to smooth consumption via saving/dissaving.

Job Search

Individuals choose search effort s_{ij} to maximize W^U_{ij} taking all "macro" variables as given. The first order conditions to the search problem read[8]

$$f'(1-s_{BB}) = \pi\alpha_B\left[W^E_{BB}-W^U_{BB}\right]+(1-\pi)\alpha_G\left[W^E_{GB}-W^U_{GB}\right] \tag{1}$$

$$f'(1-s_{BG}) = \pi\alpha_B\left[W^E_{BB}-W^U_{BB}\right]+(1-\pi)\alpha_G\left[W^E_{GB}-W^U_{GB}\right] \tag{2}$$

$$f'(1-s_{GG}) = \pi\alpha_G\left[W^E_{GG}-W^U_{GG}\right]+(1-\pi)\alpha_B\left[W^E_{BG}-W^U_{BG}\right] \tag{3}$$

$$f'(1-s_{GB}) = \pi\alpha_G\left[W^E_{GG}-W^U_{GG}\right]+(1-\pi)\alpha_B\left[W^E_{BG}-W^U_{BG}\right] \tag{4}$$

Note that search depends in the usual way on the gain from shifting from unemployment into a job. However since the business cycle situation may change, job search depends on the gain from finding a job if remaining in the current state (probability π) and the gain if there is a shift in the state of nature (probability $1-\pi$). The higher π, the more search is affected by the current state and vice versa.

It follows immediately that search depends on the current state of nature only and hence there are only two levels of search, i.e.

$$s_{BB} = s_{BG} = s_B$$

$$s_{GG} = s_{GB} = s_G$$

The intuition behind this implication is that the search decision is forward looking in the sense that current search influences the future labour market status, and therefore it is independent of the past state[9].

To see how the state of nature affects search consider first the special case where $\pi = 1$ we have

$$s_G > s_B \text{ for } \alpha_G\left[W^E_{GG}-W^U_{GG}\right] > \alpha_B\left[W^E_{BB}-W^U_{BB}\right]$$

[8] Concavity of the f function ensures that the second order conditions are fulfilled.

[9] Note that the separability assumption is crucial for this property.

that is, if the expected gain from finding a job (=job finding rate α times gain from being employed $W^E - W^U$) is larger in the good state than in the bad state, then agents search more in the good than the bad state of nature, and vice versa.

Considering next how the possibility of a change in state of nature captured by how π affects search we have

$$sign\left(\frac{\partial s_G}{\partial \pi}\right) = sign\left(\alpha_G\left[W_{GG}^E - W_{GG}^U\right] - \alpha_B\left[W_{BG}^E - W_{BG}^U\right]\right)$$
$$sign\left(\frac{\partial s_B}{\partial \pi}\right) = sign\left(\alpha_B\left[W_{BB}^E - W_{BB}^U\right] - \alpha_G\left[W_{GB}^E - W_{GB}^U\right]\right)$$

More persistence in the business cycle situation (higher π) tends to increase search effort if the expected gain from search is higher in the current state than in the new "swing" state, and vice versa.

2.2 Firms

A filled job generates an output (exogenous) y and firms can create job vacancies at a flow cost of ky $(k > 0)$. A filled job may be destroyed next period if there is a job separation. The value of a filled job in a given state of nature is

$$\rho J_B^E = y - w + \pi p_{BB}(J_B^V - J_B^E) + (1-\pi)\left[p_{GB}(J_G^V - J_B^E) + (1 - p_{GB})(J_G^E - J_B^E)\right] \tag{5}$$

$$\rho J_G^E = y - w + \pi p_{GG}(J_G^V - J_G^E) + (1-\pi)\left[p_{BG}(J_B^V - J_G^E) + (1 - p_{BG})(J_B^E - J_G^E)\right] \tag{6}$$

Note that the value of a filled job does not depend on the past state. A vacant job may be filled in the future if there is a job match, and hence the current value of a vacant job in a given state is

$$\rho J_B^V = -ky + \pi q_B(J_B^E - J_B^V) + (1-\pi)q_G(J_G^E - J_G^V)$$
$$\rho J_G^V = -ky + \pi q_G(J_G^E - J_G^V) + (1-\pi)q_B(J_B^E - J_B^V)$$

where q_i denotes the probability of filling a vacant job (see below). Vacancies are created up to the point where the value of a vacancy is zero, i.e.

$$J_G^V = J_B^V = 0$$

implying

$$0 = -ky + \pi q_B J_B^E + (1-\pi) q_G J_G^E$$
$$0 = -ky + \pi q_G J_G^E + (1-\pi) q_B J_B^E$$

From these conditions it follows that

$$\pi q_G J_G^E + (1-\pi) q_B J_B^E = \pi q_B J_B^E + (1-\pi) q_G J_G^E$$

It is an implication that

$$J_B^E = \frac{q_G}{q_B} J_G^E \tag{7}$$

i.e. the relative value of having a filled jobs in either state (B or G) depends on the ratio of the job finding rates.

$$J_G^E > J_B^E \quad \text{if} \quad q_B > q_G$$

Hence, the value of a filled job is higher in the G state than in the B state provided the job filling rate is lower $q_G < q_B$. The intuition is that the more difficult it is to fill a vacant job, the higher the value of a filled job.

The value of a filled job in the two states is therefore given as

$$J_B^E = \frac{ky}{q_B}$$
$$J_G^E = \frac{ky}{q_G}$$

2.3 Wages

Wages are assumed to be set in a Nash-bargain after a match has been made. Employed are represented by unions having the objective of maximizing wages for employed workers. As has been argued in non-cooperative approaches to justify this bargaining model the relevant outside option is what can be achieved during delay in reaching an agreement (see Binmore, Rubinstein and Wolinski (1986)). This outside option is assumed to be zero for both workers and firms and hence the wage setting problem is given as the solution to

$$Max_w \quad [w]^\beta [y-w]^{1-\beta}$$

where $0 < \beta < 1$ $(1-\beta)$ is the bargaining power of workers (firms). This wage setting model implies that the wage is given as

$$w = \beta y$$

The main attraction of this approach is that it gives a simple wage relation which in accordance with empirical evidence implies that the wage is rigid across states of nature. Alternative routes may be pursued in modelling wage rigidities (see Hall (2005) and Hall and Milgrom (2008) for recent work in a search matching context) and the specific formulation adopted here is to be considered as an illustrative workhorse model. The crucial property is that wages do not respond to variations in unemployment (job separations etc.)[10].

2.4 Public sector

The public sector provides the benefit level b_i to unemployed in a given state of nature i and finances this by a proportional tax rate and a (path dependent) lump sum tax. The income tax rate τ is assumed to be constant across states of nature, i.e. any state dependency runs via the benefit level and the lump sum tax.

The primary budget balance in any state is

$$B_{ij} = (1-u_{ij})\tau w + T_{ij} - b_i u_{ij}$$

[10] Allowing for wages to be different across states of nature may contribute to dampen unemployment variations vis lower wages in downturns and higher wages in upturns, see e.g. Coles and Masters (2007).

Hence, the debt level D in the different states are given as

$$\rho D_{BB} = b_B u_{BB} - \tau w(1 - u_{BB}) - T_{BB} + (1 - \pi)[D_{GB} - D_{BB}]$$
$$\rho D_{GB} = b_G u_{GB} - \tau w(1 - u_{GB}) - T_{GB} + \pi [D_{GG} - D_{GB}] + (1 - \pi)[D_{BG} - D_{GB}]$$
$$\rho D_{BG} = b_B u_{BG} - \tau w(1 - u_{BG}) - T_{BG} + \pi [D_{BB} - D_{BG}] + (1 - \pi)[D_{GB} - D_{BG}]$$
$$\rho D_{GG} = b_G u_{GG} - \tau w(1 - u_{GG}) - T_{GG} + (1 - \pi)[D_{BG} - D_{GG}]$$

Since the primary budget is dependent on the current state of nature nothing ensures that the debt level is stationary. A sequence of bad draws may potentially in combination with debt servicing lead to a non-sustainable debt level. To avoid this consider the following simple policy for the lump-sum tax

$$T_{BB} = b_B u_{BB} - \tau w(1 - u_{BB})$$
$$T_{GG} = b_G u_{GG} - \tau w(1 - u_{GG})$$
$$T_{BG} = b_G u_{GB} - \tau w(1 - u_{GB})$$
$$T_{GB} = b_B u_{BG} - \tau w(1 - u_{BG})$$

This policy rule is not necessarily optimal but it allows some diversification across states of nature while at the same time ensuring a stationary debt level in all states of nature (see Appendix A). Hence, it is useful to illustrate the basic mechanisms in a simple way. Clearly, more sophisticated schemes can deliver more insurance and hence the present case tends to underestimate the scope for insurance.

The policy rule outlined above implies that the primary balance is given as

$$B_{BB} = 0$$
$$B_{BG} = [b_G u_{GB} - \tau w(1 - u_{GB})] - [b_B u_{BG} - \tau w(1 - u_{BG})]$$
$$B_{GB} = [b_B u_{BG} - \tau w(1 - u_{BG})] - [b_G u_{GB} - \tau w(1 - u_{GB})]$$
$$B_{GG} = 0$$

Note that $B_{BG} < 0$ and $B_{GB} > 0$ if $u_{BG} > u_{GB}$ and/or $b_B > b_G$, i.e. there is a net transfer when the state of nature shifts from low job separations to high job separations, and vice versa. It is thus implied that there is only an across state of nature insurance mechanism when the state of nature changes but not when it persists. This may broadly be said to capture that transitory shocks can be diversified while persistent shocks can not.

It is shown in Appendix A, that this policy implies stationary debt levels and thus satisfy the no-Ponzi condition.

2.5 Matching

Matches are determined by a standard constant returns to scale matching function, i.e. the number of matches in state i are given as

$$m(S_{ij}, V_{ij}) \equiv AS_{ij}^{\alpha}V_{ij}^{1-\alpha} \quad , 0 < \alpha < 1$$

where V_i is the number of vacancies in state i and aggregate search is given as

$$S_{ij} = s_i u_{ij}$$

The job finding rate is therefore

$$\alpha_{ij} \equiv \frac{m(S_{ij}, V_{ij})}{S_{ij}} = m(1, \theta_{ij}) = A\theta_{ij}^{1-\alpha}$$

where $\theta_{ij} \equiv \frac{V_{ij}}{S_{ij}}$ measures market tightness, and $\alpha(\theta_{ij})$, $\alpha'_{\theta}(\theta_{ij}) > 0$.

Firms fill vacancies at the rate

$$q_{ij} \equiv \frac{m(S_{ij}, V_{ij})}{V_{ij}} = m(\theta_{ij}^{-1}, 1) = A\theta_{ij}^{-\alpha}$$

where $q'_{\theta}(\theta) < 0$.

2.6 Inflow and outflow

The unemployment rate is a stock variable displaying inertia due to the matching friction. Hence in general the unemployment rate adjusts sluggishly to changes in the state of nature[11] and therefore

[11] See e.g. Pissarides and Mortensen (1994) and Shimer (2005) for business cycle versions of the search model in which the unemployment rate evolves from the initial unemployment rate conditional on the shocks.

it displays path dependence. To avoid complexities associated with this it is assumed that job separation rates differ at state transitions so as to ensure that the unemployment rate only takes on two values u_B and u_G. The intuition is that if there is a shift from the "good" to the "bad" state there is an extraordinary high job separation rate, and vice versa when shifting from a "bad" to a "good" equilibrium. Hence

$$u_{BG} = u_{BB} = u_B$$
$$u_{GB} = u_{GG} = u_G$$

The change in unemployment is given as the difference between job separations and hires. Hence, a shift between two levels of unemployment u_B and u_G is ensured for given exogenous job separation rates p_{BB} and p_{GG}, provided that the following restrictions are met

$$0 = (1 - u_B)p_{BB} - \alpha_B s_B u_B \tag{8}$$
$$u_G - u_B = (1 - u_B)p_{GB} - \alpha_G s_G u_G \tag{9}$$
$$u_B - u_G = (1 - u_G)p_{BG} - \alpha_B s_B u_B \tag{10}$$
$$0 = (1 - u_G)p_{GG} - \alpha_G s_G u_G \tag{11}$$

Note that α and s only depend on the current state and u_i $(i = B, G)$ is the stationary unemployment rate if there is no change in the state of nature. It is an implication that the above conditions determine p_{GB} and $p_{BG.}$[12]. From (8) and (10) we have

$$p_{BG} = \frac{u_B - u_G}{(1 - u_G)} + \frac{(1 - u_B)}{(1 - u_G)} p_{BB} \tag{12}$$

and from (9) and (11) that

$$p_{GB} = \frac{u_G - u_B}{(1 - u_B)} + \frac{(1 - u_G)}{(1 - u_B)} p_{GG} \tag{13}$$

It follows that $u_G - u_B < 0$ implies that a shift from an G-state to a $B-$state is associated with extraordinary high job separations, i.e. $p_{BG} > p_{BB}$ and a shift from a B-state to a G-state is associated with an extraordinary low level of job separations[13], i.e. $p_{GB} < p_{GG}$.

[12] Note that this makes the job separations at "switching" states a jump variable, to ensure that unemployment only varies between two levels.

[13] Conditions ensuring that $p_{GB} > 0$ are assumed fulfilled.

3 Stationary state

As a point of reference consider the case where the state of nature is invariant, i.e. there is no shift in state of nature ($\pi = 1$) or alternatively that the job separation rate is constant ($p_{BB} = p_{GG} = p$). In this case there is a stationary equilibrium with a given unemployment rate u and the budget balances.

3.1 Optimal benefit level: incentives vs. insurance

The optimal benefit level involves a trade-off between insurance and incentives. To clarify this a utilitarian objective function is assumed. While this is not unproblematic it is useful in the present context since it allows a focus on the crucial trade-off arising. Under this assumption the objective function of the social planner is

$$\Psi \equiv (1-u)\, W^E + uW^U \tag{14}$$

where the policy maker takes into account that[14]

$$\rho W^E = h(w(1-\tau)) + e(1-l) + p\left[W^U - W^E\right]$$
$$\rho W^U = g(b) + f(1-s) + \alpha s\left[W^E - W^U\right]$$

and that search is determined by

$$f'(1-s) = \alpha\left[W^E - W^U\right]$$

The budget constraint reads

$$(1-u)\,\tau w = ub$$

The first order condition to the problem of finding the optimal benefit level maximizing (14) can now be written

$$(1-u)\,\frac{\partial W^E}{\partial b} + u\frac{\partial W^U}{\partial b} + \frac{\partial u}{\partial b}\left[W^U - W^E\right] = 0$$

[14] The lump sum tax is set equal to zero, $T = 0$, without loss of generality.

or in more compact form as[15][16]

$$g_c(b) - h_c(w - \frac{u}{1-u}b) = \Lambda \frac{\partial u}{\partial b} \qquad (15)$$

where

$$\Lambda \equiv \frac{1}{u}\frac{f'(1-s)}{\alpha} > 0$$

To interpret (15) note first that if $\frac{\partial u}{\partial b} = 0$ (no incentive effects of unemployment benefits) we have that optimal benefits are determined by the condition

$$g_c(b) = h_c(w - \frac{u}{1-u}b) \qquad (16)$$

i.e. the optimal benefit level is where the marginal utility of income is the same for employed and unemployed[17]. This is known as the "Borch condition" for full insurance (Borch (1960)). The insurance effect is not directly related to the unemployment rate but depends on the conditions prevailing as either unemployed or employed. However, there is a budget effect since the benefits are financed by taxes levied on the employed and we have

$$\frac{db}{du} = -\frac{h_{cc}(w - \frac{u}{1-u}b)\frac{b}{(1-u)^2}}{g_{cc}(b) + h_{cc}(w - \frac{u}{1-u}b)\frac{u}{1-u}} < 0$$

i.e. a higher unemployment rate is accompanied by lower benefits. The intuition is that higher unemployment raises the financing requirements for a given benefit level, which in turn reduces the

[15]Note that $(1-u)W^E + uW^U = (1-u)\left[h(w - \frac{u}{1-u}b) + e(1-l)\right] + u\left[g(b) + f(1-s)\right]$ hence

$$(1-u)\frac{\partial W^E}{\partial b} + u\frac{\partial W^U}{\partial b} = u\left[g_c(b) - h_c(w - \frac{u}{1-u}b)\right].$$

[16]Using that $p(1-u) = \alpha s u$.

[17]Note that the participation constraint is implicitly assumed fulfilled. Otherwise there is an additional constraint in which case the benefit level is determined by the "corner" condition that

$$\left[h(w - \frac{u}{1-u}b) - e(1-l)\right] - \left[g(b) - f(1-s_u)\right] = 0$$

disposable income of employed and thus raise their marginal utility of income. To rebalance the marginal utility of consumption between the two groups it is necessary to lower benefits. While non-distortionary benefits is a special case, this points out an important effect arising under a balanced budget requirement.

Returning to the general expression for optimal benefits given in (15), it has a straightforward interpretation. The LHS gives the value of insurance measured by the difference in marginal utility of income between unemployed and employed. The RHS gives the consequences of the effects of benefits on incentives measured by the effect on the unemployment rate. The condition therefore says that the value of insurance - the LHS - should equal the costs of insurance in terms of distorted incentives - the RHS. When higher benefits lead to higher unemployment ($\frac{\partial u}{\partial b} > 0$) (the incentive effect) the RHS of (15) is positive and it is implied that

$$g_c(b) - h_c(w - \frac{u}{1-u}b) > 0$$

i.e. taking the incentive effect into account it is not optimal to provide full insurance. Therefore the marginal utility of income of unemployed is larger than that of the employed. The larger the distortion measured by the numerical value of $\frac{\partial u}{\partial b}\Lambda$, other things being equal, the lower the benefit level and hence the larger $g_c(b) - h_c(w - \frac{u}{1-u}b)$. The point is that the optimal insurance provided depends negatively on the incentive effect - the more unemployment benefits create disincentives, the lower the optimal benefit level. Oppositely, this also implies that one cannot conclude from the fact that benefits lead to higher unemployment, that benefits are too high - the insurance effect has to be taken into account.

Finally note that the condition (15) determining the optimal benefit level alternatively may be written[18]

$$\frac{\left[g_c(b) - h_c(w - \frac{u}{1-u}b)\right] b}{[W^E - W^U]} = \frac{\partial u}{\partial b}\frac{b}{u} \tag{17}$$

This shows that the marginal costs of providing insurance (the RHS of (17)) can be expressed in

[18] Note that

$$W^E - W^U = \frac{h(w-T) + e(1-l) - g(b) - f(1-s)}{\rho + p + \alpha s}$$

terms of the elasticity of unemployment wrt. the benefit level.

3.2 Optimal benefits and unemployment

To see how the optimal benefit level depends on the underlying characteristics of the economy consider variations in the job separation rate[19], that is, we compare stationary equilibria for different levels of the job separation rate. We report the results of some illustrative simulations of the model.

The simulations are conducted assuming the following functional forms:

$$
\begin{aligned}
h\left(w\left[1-t\right]-T_{ij}\right) &= \frac{\left(w\left[1-t\right]-T_{ij}\right)^{1-i}}{1-i} \\
g\left(b_i-T_{ij}\right) &= \frac{\left(b_i-T_{ij}\right)^{1-i}}{1-i} \\
f\left(1-s_i\right) &= \ln\left(1-s_i\right) \\
e\left(1-l\right) &= \ln\left(1-l\right)
\end{aligned}
$$

with $i = 8$.

Following among others Frederiksson & Holmlund (2006), the matching function is assumed to be Cobb-Douglas of the form $m = As^{1-\alpha}v^{\alpha}$, with $\alpha = 0.5$ and $A = 0.29$. Time is quarterly and we discount utility at $\rho = 0.003$ and assume that workers spend 10% of their time at work, $l = 0.1$. The tax rate is $t = 0.01$ and $\beta = 0.9$. Finally, output is set to $y = 1$, vacancy costs are set to $k = 0.2$.

For each level of the job separation rate, we report the unemployment rate and the optimal benefit level (cf. (15)).

[19] Differences in the business cycle situation may be generated by changing other variables in the model like job creation, the costs of vacancies, matching efficiency etc., but the qualitative results would be the same, see Andersen and Svarer (2008).

Figure 1: One state model: unemployment and net compensation to unemployed

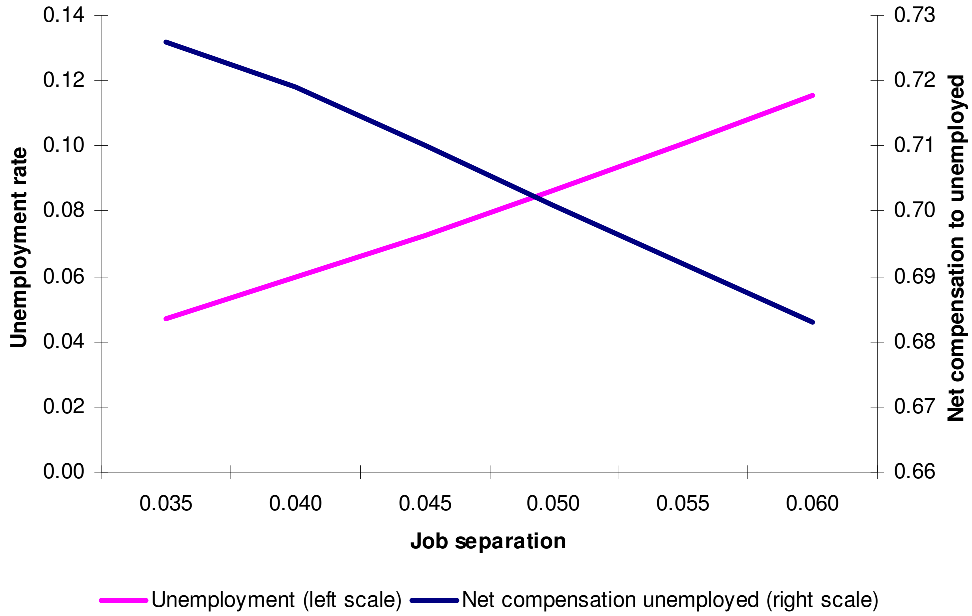


Figure 1 shows as expected that equilibrium unemployment is higher, the higher the job separation rate. The optimal benefit level (net compensation) is seen to be decreasing in the job separation rate and thus falling in the unemployment rate. Hence, in the one state case optimal benefits are pro-cyclical; if unemployment is high, net compensation is low, and vice versa. The main driver behind this is the budget effect discussed above.

The basic lesson from the one state model is thus that the budget effect arising via the balanced budget requirement plays an important role and it tends to imply pro-cyclical benefit levels. The incentive effects or distortions are also important and an important question is whether these vary across states of nature and if so whether this can lead to counter-cyclical benefit levels being optimal. We address this question in the following section.

4 Shifting states of nature and business cycles

We turn now to the case where the state of nature may change, that is, the economy is exposed to business cycle fluctuations. The nature of the cycle is such that there are shifts between "good" and "bad" states, and the business cycle situation remains unchanged with probability π and changes with probability $1-\pi$ $(0<\pi<1)$. The probability π is therefore also a measure of the persistence in the business cycle situation.

In Appendix B it is proved that there exists a unique equilibrium in which $\theta_G > \theta_B$ implying that i) unemployment is higher in a bad state than a good state, i.e. $u_B > u_G$, ii) the job finding rate is lower in a bad state $\alpha_B < \alpha_G$, iii) the job filling rate is higher in a bad state $q_B > q_G$, and therefore iv) the value of a filled job is higher in a good state $J_G^E > J_B^E$.

4.1 Shifting business cycle conditions and search

The possibility of changes in the business cycle situation affects search behaviour since unemployed take into account that the state of nature may change. To see how changes in the business cycle affect job search consider the determination of job search from (11) here repeated for search in the bad state for the sake of argument,

$$f'(1-s_{BB}) = \pi\alpha_B \left[W_{BB}^E - W_{BB}^U\right] + (1-\pi)\alpha_G \left[W_{GB}^E - W_{GB}^U\right]$$

If there is no possibility for a change to the "good" state $(\pi = 1)$ we have that the RHS equals

$$\alpha_B \left[W_{BB}^E - W_{BB}^U\right]$$

Now suppose that $W_{BB}^E - W_{BB}^U = W_{GB}^E - W_{GB}^U$ then a possibility of shifting to the "good" state $(0<\pi<1)$ will increase search in the "bad" state since

$$\pi\alpha_B + (1-\pi)\alpha_G > \alpha_B \text{ for all } \pi < 1 \text{ if } \alpha_G > \alpha_B$$

i.e. the possibility of a shift to a state with a higher job finding rate increases other things being equal the search level, and the effect is stronger, the larger the difference in job finding rates between the two states.

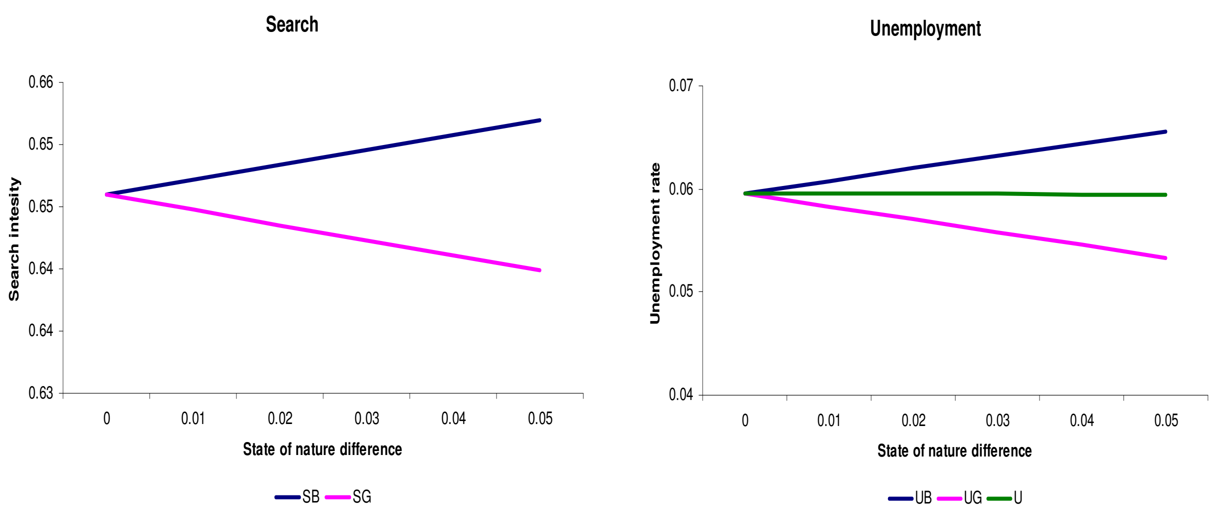
The effect is obviously the opposite for search in the good state of nature, i.e.

$$\pi\alpha_G + (1-\pi)\alpha_B < \alpha_G \text{ for all } \pi < 1 \text{ if } \alpha_G > \alpha_B$$

Hence, we have that business cycle fluctuations tend to induce more search in the bad state, and less search in the good state. The mechanism driving this is the difference in job finding rates in combination with the possible change in the business cycle situation.

These effects are illustrated in Figure 2 showing on the x axis the widening of the difference in the job separation rate between the two states of nature (zero difference corresponds to a one state model). It is seen that job search is higher in bad states. The difference widens as expected as the two states become more different. For the unemployment rate it is as expected the case that unemployment is higher in the bad and lower in the good state. Note that the average unemployment rate is (slightly) decreasing as the difference widens, that is, the unemployment rate is convex in the job separation rate (see also Hairault et. al. (2008)) and therefore business cycle fluctuations affect the structural/average unemployment rate.

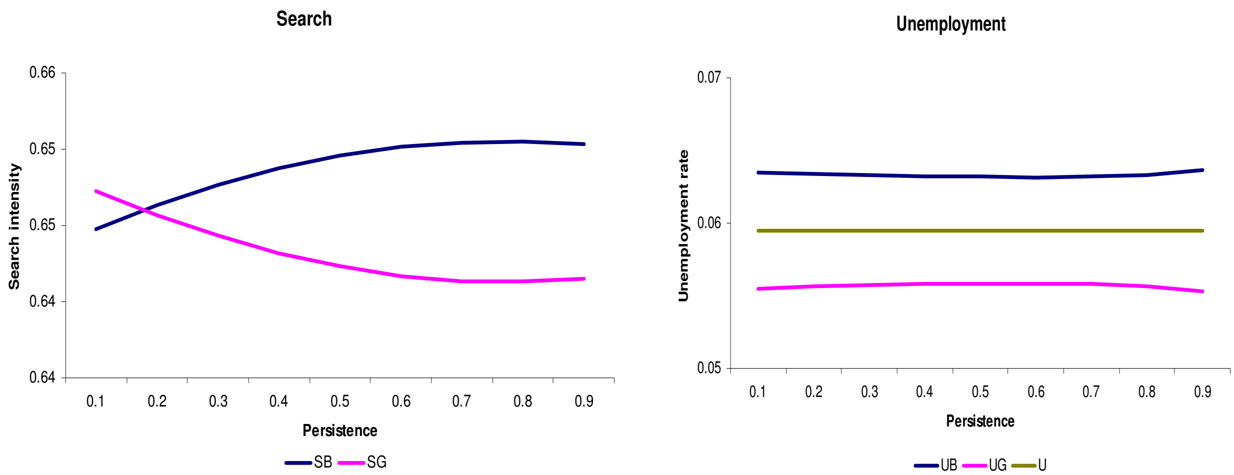
Figure 2: Widening business cycle differences: Search and unemployment



Note: For 0 the job separation rates are $p_{BB} = p_{GG} = 0.04$, and for each step 0.01 is added to p_{BB} and subtracted from p_{GG},and the persistence is $\pi = 0.5$.

The reasoning given above also suggests that the persistence in the business cycle situation (measured by π) matters since it influences expectations. The larger π the more expectations are anchored in the current state, and vice versa. Intuitively if persistence is weak, expectations are driven by the situation in the alternate state, and oppositely if persistence is strong. This is also seen from Figure 3 showing that there is a critical level of persistence above which search is largest in the bad state. It is a consequences that unemployment rates differ slightly more between the two states of nature if π is either low or high.

Figure 3: Persistence in business cycle situation: search and unemployment



Note: here $p_{GG} = 0.042$ and $p_{BB} = 0.038$.

4.2 Distortions

The analysis in section 3 pointed to the distortionary effects of benefits levels on unemployment as being crucial for the optimal benefit level (see e.g. (17)). Intuitively, the benefit level should be more distortionary in good states of nature with higher job finding rates than in bad states of nature. The following tables consider this issue and reports the elasticities of search and unemployment, respectively, with respect to the benefit level in the two possible states of nature. Consider first search. As expected higher benefits lower search. There is both a direct effect in the state of nature for which the change applies, but also an effect in the alternate state since agents perceive business cycle fluctuations. If the business cycle situation is sufficiently persistent the direct effect is larger than the indirect effect in the alternate state of nature. Most importantly it is seen that in all cases the direct effect is numerically larger in the good than in the bad state, i.e. search is affected more by benefits in good than in bad states of nature.

Table 1: Effects of changing benefits: elasticity of search intensity wrt. benefit level

	$\pi = 0.3$		$\pi = 0.5$		$\pi = 0.7$	
	b_B	b_G	b_B	b_G	b_B	b_G
Percentage change in search intensity, bad state: s_B	-0.83	$-1\ .67$	-1.17	-1.29	-1.58	-0.87
Percentage change in search intensity, good state: s_G	-1.60	$-0\ .92$	-1.29	-1.26	-0.90	-1.69

Note: $p_{BB} = 0.042$ and $p_{GG} = 0.038$.

Second, similar effects are found for unemployment, that is, unemployment increases when benefits are increased. Also here there is a direct effect which is stronger the more persistent the business cycle situation, whereas the indirect effect on the alternate state is stronger the less persistent the business cycle situation. It is seen that direct effect of benefit increases is larger in good than in bad states of nature, i.e. the distortions are business cycle dependent and we have that they are larger in good than in bad states. This goes in the direction of making optimal benefit levels state dependent, and we explore issue in the next section.

Table 2: Effects of changing benefits: elasticity of unemployment rate wrt. benefit level

	$\pi = 0.3$		$\pi = 0.5$		$\pi = 0.7$	
	b_B	b_G	b_B	b_G	b_B	b_G
Percentage change in unemployment, bad state: u_B	0.80	1.56	1.10	1.21	1.47	0.83
Percentage change in unemployment, good state: u_G	1.52	0.80	1.24	1.21	0.88	1.61
Percentage change in mean unemployment:u	1.12	1.27	1.16	1.21	1.20	1.18

Note: p_{BB}= 0.045 and p_{GG}= 0.035.

4.3 State dependent benefits and insurance

Turning to the insurance aspects there are two dimensions of insurance. One is between the employed and unemployed in a given state of nature. The other dimension is across states of nature. To see this note that disposable income for the employed is

$$w(1-\tau)-T_{BB}=w-(b_B+\tau w)\,u_{BB}$$
$$w(1-\tau)-T_{BG}=w-(b_G+\tau w)\,u_{GB}$$
$$w(1-\tau)-T_{GB}=w-(b_B+\tau w)\,u_{BG}$$
$$w(1-\tau)-T_{GG}=w-(b_G+\tau w)\,u_{GG}$$

and for the unemployed

$$b_B-T_{BB}=b_B+\tau w-(b_B+\tau w)u_{BB}$$
$$b_B-T_{BG}=b_B+\tau w-(b_G+\tau w)u_{GB}$$
$$b_G-T_{GB}=b_G+\tau w-(b_B+\tau w)u_{BG}$$
$$b_G-T_{GG}=b_G+\tau w-(b_G+\tau w)u_{GG}$$

It is seen that in a given state, say BB, the disposable income of both employed and unemployed depends on the benefit level in combination with the unemployment rate. By changing the benefit level it is thus possible to provide insurance (redistribute) between employed and unemployed[20]. Second, by running a non-balanced budget in the swing states (GB and BG) it is possible to insure across states of nature. In the present context this possibility arises when the state of nature changes, and it is seen that for $b_B > b_G$ and $u_B > u_G$ that both employed and unemployed are compensated when the state shifts from G to B and vice versa. The latter is also seen by considering how a change in the state of nature affects the overall position of employed where we have

$$\rho\left[W^E_{BG}-W^E_{BB}\right]=h(w\left[1-\tau\right]-T_{BG})-h(w\left[1-\tau\right]-T_{BB})$$
$$\rho\left[W^E_{GB}-W^E_{GG}\right]=h(w\left[1-\tau\right]-T_{GB})-h(w\left[1-\tau\right]-T_{GG})$$

Hence, if $T_{BB} > T_{BG}$, and $T_{GB} > T_{GG}$ it follows that

$$W^E_{BG} > W^E_{BB}$$
$$W^E_{GB} < W^E_{GG}$$

[20] It is easily verified that it is not possible with the state dependent policy to achieve complete insurance as defined by the Borch condition for employed and unemployed across the four different possible states of nature.

Similarly the situation for the unemployed when the state of nature is given as

$$\rho\left[W_{BG}^{U}-W_{BB}^{U}\right] = g(b_H - T_{BG}) - g(b_B - T_{BB}) + W_{BB}^{U} - W_{BG}^{U}$$
$$(\rho+1)\left[W_{BG}^{U}-W_{BB}^{U}\right] = g(b_H - T_{BG}) - g(b_B - T_{BB})$$

and

$$\rho\left[W_{GB}^{U}-W_{GG}^{U}\right] = g(b_G - T_{GB}) - g(b_G - T_{GG}) + W_{GG}^{U} - W_{GB}^{U}$$
$$(\rho+1)\left[W_{GB}^{U}-W_{GG}^{U}\right] = g(b_G - T_{GB}) - g(b_G - T_{GG})$$

and if $T_{BB} > T_{BG}$ and $T_{GB} > T_{GG}$ it follows that

$$W_{BG}^{U} > W_{BB}^{U}$$
$$W_{GB}^{U} < W_{GG}^{U}$$

Intuitively the need for insurance is larger in bad than in good states. The present framework captures this. To see this note first that the instantaneous utility of benefits are the same for unemployed in any state $g(b)$. But the expected duration of unemployment captured by the job finding rate α is also of importance. If the job finding rate is high the consequences of being unemployed are less severe than if it is low, and therefore benefits matter less. To see this, it is most easy to revert to the one state case ($\pi = 1$) where we have

$$[\rho + \alpha s]\, W^{U} = g(b) + f(1-s) + \alpha s W^{E}$$

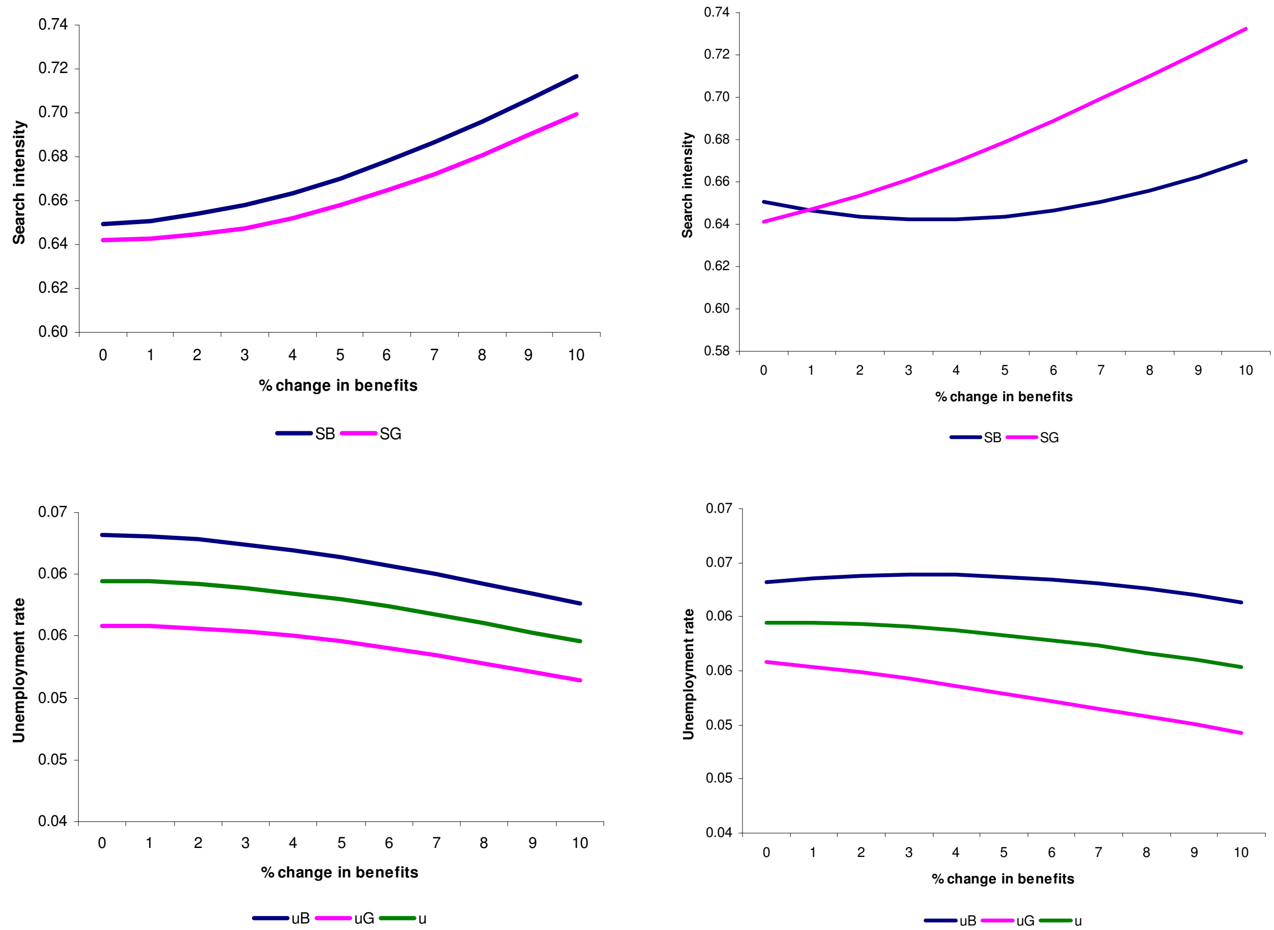
which implies

$$\frac{\partial W^{U}}{\partial b} = \frac{g_c(b)}{\rho + \alpha s} > 0$$
$$\frac{\partial}{\partial \alpha}\frac{\partial W^{U}}{\partial b} < 0$$

Hence, other things being equal a lower job finding rate (α) increases the marginal value of benefits. The reason is straightforward, the lower the job finding rate, the longer is the expected duration of an unemployment spell, and hence the more valuable it is to have high benefits.

Figure 4: State dependent benefits: search and unemployment depending on persistence

$\pi = 0.5$ $\pi = 0.7$



Note: "% change in benefits" gives the increase in benefits in the bad state and decrease in the good state relative to the initial case (0) where the benefit level is state independent. Hence, the span between the two benefits levels is two times "% change in benefits".

Figure 4 shows the consequences of state contingent benefits for two levels of persistence ($\pi = 0.5$ and 0.7) in the business cycle situation. The figure reports on the x-axis the % increase (decrease) in the benefit level in the bad (good) state relative to an initial situation with state independent benefits. Higher benefits in the "bad" state and lower in the "good" state provide more insurance. Intuitively it may be expected that this unambiguously would lead to less search in the "bad" state and more search in the "good" state. One striking finding is that moving from state independent to state dependent benefits may increase job search in both states of nature. To see this consider the case where $\pi = 0,5$. That search increases in the G state where benefits are reduced is straightforward, it is more surprising that it also increases in the B state where benefits are increased. To see the reason for this note that search in the B state is determined by

$$f'(1 - s_{BB}) = \pi\alpha_B \left[W_{BB}^E - W_{BB}^U\right] + (1 - \pi)\alpha_G \left[W_{GB}^E - W_{GB}^U\right]$$

The RHS gives the marginal gain from search as the probabilities of being in the various states times the job finding rate and times the gains from becoming employed. Consider now a case where $\pi = 1 - \pi = 1/2$ where we decrease $\left[W_{BB}^E - W_{BB}^U\right]$, and increase $\left[W_{GB}^E - W_{GB}^U\right]$ but under the constraint that $\left[W_{BB}^E - W_{BB}^U\right] + \left[W_{GB}^E - W_{GB}^U\right] = constant$. Then the RHS increases if $\alpha_G > \alpha_B$, i.e. since the job finding rate is higher in the G state the change in the gains from finding employment in that state matters more, ceteris paribus, than the change in the gain in the B state.

Obviously the strength of this effect depends on the persistence in the business cycle situation. As seen from the figure if the business cycle situation is reasonable persistent ($\pi = 0.7$) we have that search in the good state unambiguously increases when the benefit level is lowered. In the bad state higher benefits may first lead to lower search but for larger increases it leads to more search. The reason is that the expected gain from shifting to the good state is lower here due to the higher persistence.

Interestingly state dependent benefits work to lower average (structural) unemployment, see Figure 4. However, the implication for unemployment fluctuations are ambiguous. If the business cycle situation is not very persistent ($\pi = 0.5$) we have that the divergence in unemployment across the two states narrows and hence unemployment variability falls. If the business cycle situation is more persistent ($\pi = 0.7$) the divergence widens and unemployment variability goes up. It is thus in general ambiguous whether state dependent benefits lead to more or less unemployment variability even if the structural unemployment rate falls.

4.4 Optimal state dependent benefits

Turning now to the issue of optimal asymmetry in benefits between the two states assessed from the utilitarian criterion. In the general case we have that total utility can be written

$$\Psi = \sum_{i,j=B,G} q_{ij} \left[(1-u_{ij}) W_{ij}^E + u_{ij} W_{ij}^U \right]$$

where q_{ij} is the probability of being in state (i, j). Solving for the optimal benefit levels (b_B and b_G) we have the following first order conditions

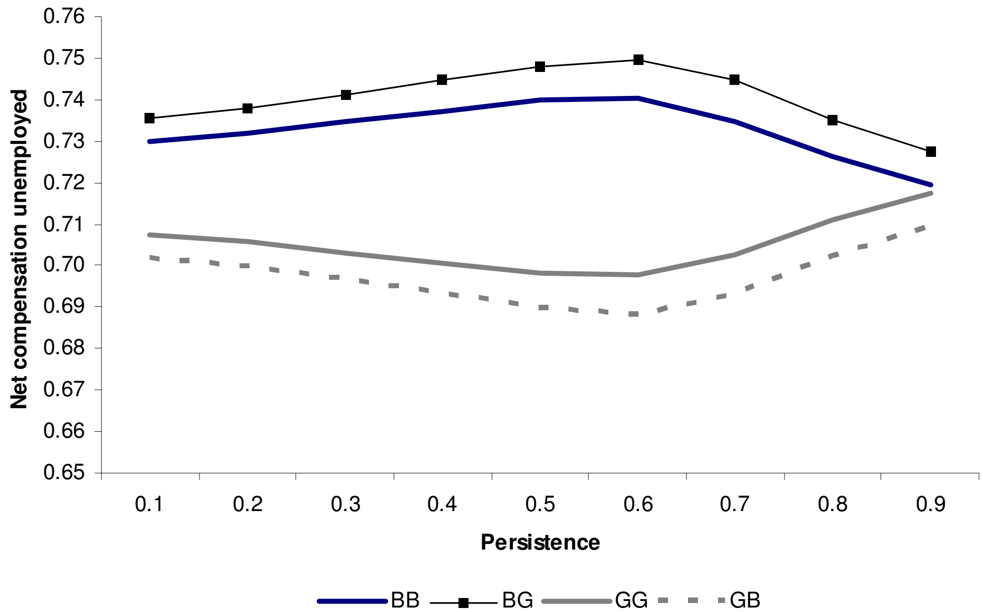
$$\sum_{i,j=B,G} q_{ij} \left[(1-u_{ij}) \frac{\partial W_{ij}^E}{\partial b_k} + u_{ij} \frac{\partial W_{ij}^U}{\partial b_k} + \left[W_{ij}^E - W_{ij}^U \right] \frac{\partial u_{ij}}{\partial b_k} \right] = 0 \text{ for } k = B, G \tag{18}$$

Which is an obvious generalization of (15).

Figure 5 below shows how the optimal net compensation (benefits less taxes paid) for the four possible states of nature depends on the underlying persistence in the business cycle situation[21]. It is seen that the net compensation is highest when a bad state follows a good state, and the intuition is that unemployed are compensated for the more bleak outlooks and lower possibilities of finding a job. Oppositely we have the lowest net compensation when a good state follows a bad state. The net compensation offered when the bad state persists (BB) is higher than when the good state persists (GG). It is seen that the differences in net compensation are largest for intermediary levels of persistence. The intuition is that the expected gains from shifting status become lower in bad states and higher in good states of nature.

[21] We present the optimal net compensation imposing a symmetry condition, that is, increase in bad states equals decreases in good states. Considering whether optimal policies imply asymmetric adjustments we found only small differences to the symmetric case.

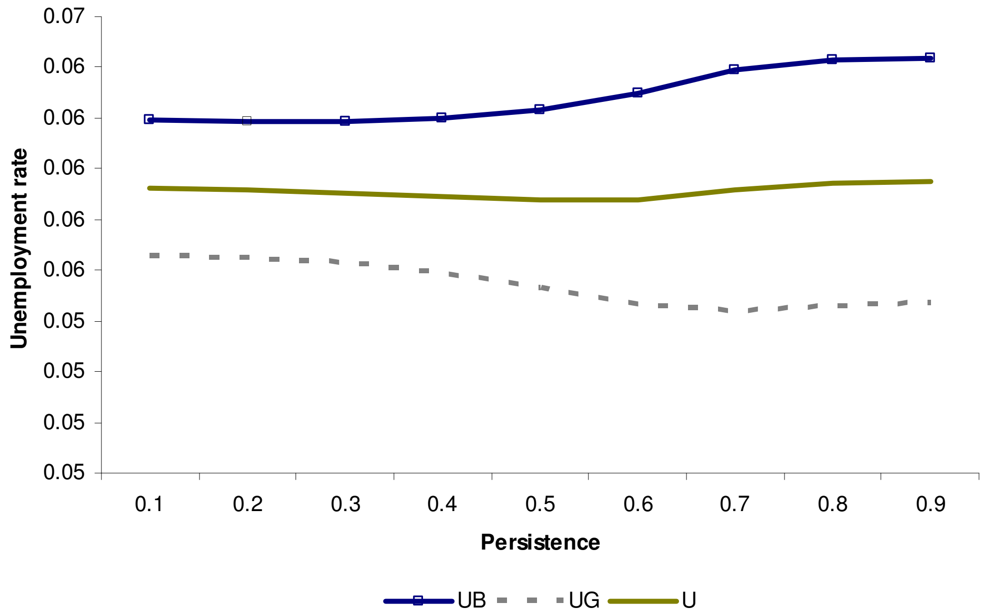
Figure 5: State dependent net compensation to unemployed and persistence



Note: The net compensation is given as $b_i - T_{ij}$. The optimal level is found in the class of symmetric state dependencies in benefit levels, i.e. the increase in the bad state equals the decrease in the good state.

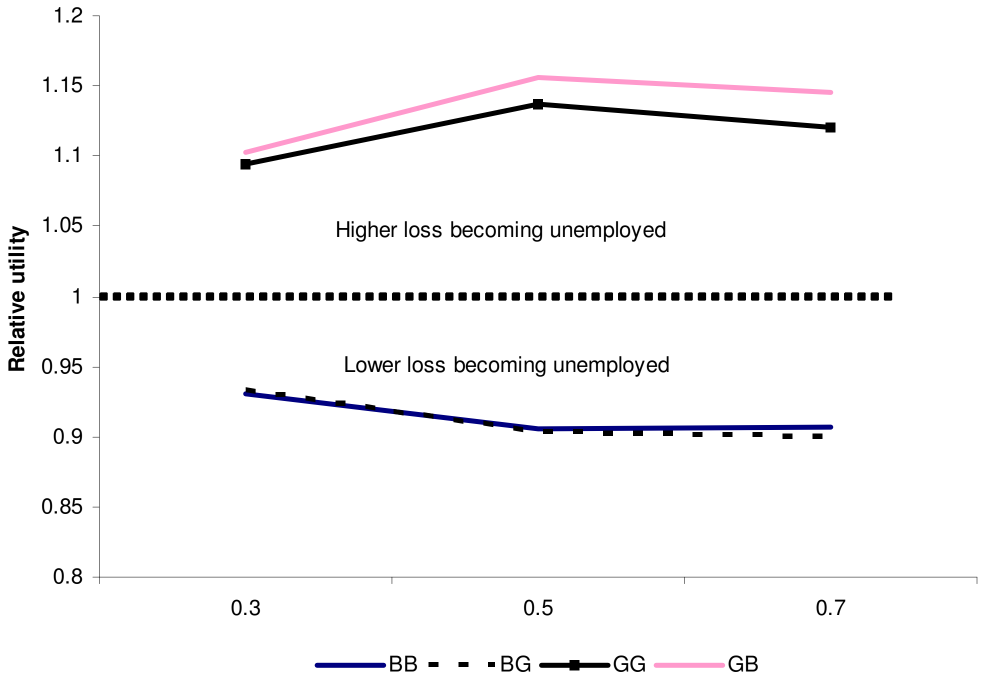
The paths for the net compensation of unemployed are reflected in the unemployment rates in the two states of nature, and thus the average (structural) unemployment rate. Unemployment is higher in bad states and lower in good states, and the difference is widening with the persistence of the business cycle situation. The average (structural) unemployment rate is for the case considered weakly U-shaped in the persistence of the business cycle situation.

Figure 6: State dependent benefits: unemployment and persistence



One way to see the welfare consequences of state dependent benefit levels is given in Figure 7. It shows that the optimal policy implies that the consequences of becoming unemployed in good states implies a larger utility loss than if benefits were state independent. In bad states the welfare loss from becoming unemployed is lowered. In this way one may say that the optimal state contingent policy effectively transfers utility from good to bad states of nature.

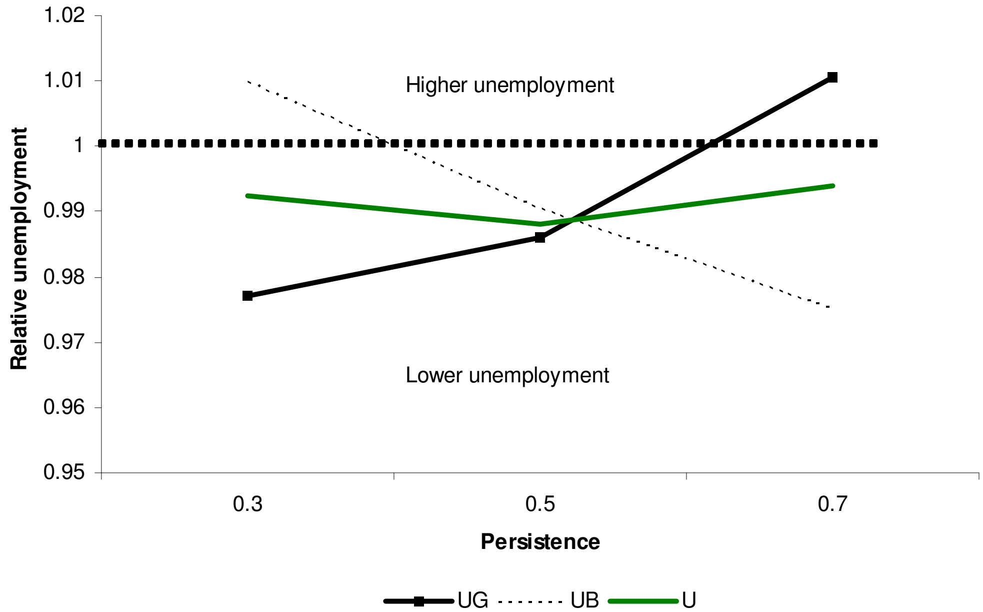
Figure 7: Utility consequences of becoming unemployed - Constant vs. state dependent benefits



Note: The figure show the utility of state dependent benefits relativ to the utility flow in a model with state independent benefits. The utility in the latter model is normalized to 1.

Finally, note that the welfare consequences differ from the consequences on the unemployment rate. Figure 8 shows that optimal state dependent benefits imply more variability in unemployment rates than state independent benefits. The reason is that benefits are increased in bad times with high unemployment, and decreased in good times with low employment. Hence, insurance shifts compensation from good to bad times, and search effort from bad to good times. In this way insurance and incentives are better aligned with the business cycle situation. An implication of this is that the average unemployment rate is lower.

Figure 8: Unemployment: Constant vs state dependent benefits



Note: The figure show the unemployment with state dependent benefits relativ to the level of unemployment in a model with state independent benefits. The level of unemployment in the latter model is normalized to 1.

This shows that it is possible to improve the insurance properties by making benefit levels state dependent without causing an increase in structural (average) unemployment rate. However, this gains may be achieved at the cost of more variability in unemployment.

5 Conclusion

Making benefit levels dependent on the business cycle situation has been shown to depend not only on an insurance effect but also a budget and an incentive (distortion) effect. The budget effect tends to make benefit levels pro-cyclical since there are higher benefit expenditures in bad times with high unemployment, and vice versa. Hence, counter-cyclical benefit levels can only arise if the incentive effects of unemployment benefits are state contingent. We have shown in a stylized business cycle model that if benefits distorts more in good than in bad times this gives an argument for having state dependent benefit levels offering higher compensation in bad than in good times. It is an important implication that while such a dependency is welfare improving by shifting utility from good to bad times, this tends to reduce structural (average) unemployment, but it may imply that the unemployment rate become more sensitive to the business cycle situation. The present analysis therefore shows that a state dependent unemployment insurance system can provide better insurance with out resulting in higher structural unemployment.

The present analysis has present a very stylized form of an unemployment insurance scheme focussing entirely on the benefit level. In practive the benefit duration may be a more important dimension of the unemployment insurance scheme to change depending on the business cycle situation. We conjecture that the case for such a state dependency is qualitatively the same as the one found in this paper for the benefit level.

The are many possible extensions of the current analysis. First, we completely ignore aggregate demand side effects of runing a state dependent policy. In relation to the automatic stabilizer effect of unemployment insurance future research expanding this dimension might provide more support for having a state dependent policy. Second, the model used in this paper relies on a very stylized description of the business cycle and a somewhat rudimentary policy rule for diversification across states of nature. It would be interesting to extent the model in these two dimensions. Something which we leave for future work.

Appendix A: Stationary debt levels

To see that this policy rules ensures stationary debt levels in all states note that the primary budget balance now can be written

$$
\begin{aligned}
B_{BB} &= 0 \\
B_{BG} &= [b_G u_{GB} - \tau w(1 - u_{GB})] - [b_B u_{BG} - \tau w(1 - u_{BG})] \\
B_{GB} &= [b_B u_{BG} - \tau w(1 - u_{BG})] - [b_G u_{GB} - \tau w(1 - u_{GB})] \\
B_{GG} &= 0
\end{aligned}
$$

implying

$$B_{BG} = -B_{GB}$$

i.e. if the public sector is running a budget deficit when a bad state of nature with high job separations ($B_{BG} < 0$) replacing a good state of nature with low job separations, then it will run a similar surplus when a good state of nature replaces a bad state of nature. In this way the scheme allows some risk diversification. To see that this is consistent with a stationary debt level in any state of nature observe further that

$$
\begin{aligned}
\rho D_{GB} &= b_G u_{GB} - \tau w(1 - u_{GB}) - [b_B u_{BG} - \tau w(1 - u_{BG})] \\
&\quad + \pi [D_{GG} - D_{GB}] + (1 - \pi) [D_{BG} - D_{GB}] \\
\rho D_{BG} &= b_B u_{BG} - \tau w(1 - u_{BG}) - [b_G u_{GB} - \tau w(1 - u_{GB})] \\
&\quad + \pi [D_{BB} - D_{BG}] + (1 - \pi) [D_{GB} - D_{BG}]
\end{aligned}
$$

implying that

$$(\rho + \pi) [D_{GB} + D_{BG}] = \pi [D_{GG} + D_{BB}]$$

and since we have from the debt level equation for D_{GG} and D_{BB} that

$$(\rho + 1 - \pi) [D_{GG} + D_{BB}] = (1 - \pi) [D_{GB} + D_{BG}]$$

we can show that

$$
\begin{aligned}
D_{GB} + D_{BG} &= 0 \\
D_{GG} + D_{BB} &= 0.
\end{aligned}
$$

$$\rho D_{BB} = b_B u_{BB} - \tau w(1-u_{BB}) - T_{BB} + (1-\pi)\left[D_{GB} - D_{BB}\right]$$
$$\rho D_{GB} = b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB} + \pi\left[D_{GG} - D_{GB}\right] + (1-\pi)\left[D_{BG} - D_{GB}\right]$$

$$\rho D_{BB} = b_B u_{BB} - \tau w(1-u_{BB}) - T_{BB} + (1-\pi)\left[D_{GB} - D_{BB}\right]$$
$$\rho D_{GB} = b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB} + \pi\left[-D_{BB} - D_{GB}\right] - 2(1-\pi)D_{GB}$$

$$(\rho + 1 - \pi)\, D_{BB} = (1-\pi)D_{GB}$$
$$(\rho + \pi + 2(1-\pi))\, D_{GB} = b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB} - \pi D_{BB}$$

$$(\rho + \pi + 2(1-\pi))\, D_{GB} = b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB} - \pi\frac{1-\pi}{\rho+1-\pi}D_{GB}$$
$$D_{GB} = \left[\rho + \pi + 2(1-\pi) + \pi\frac{1-\pi}{\rho+1-\pi}\right]\left[b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB}\right]^{-1}$$

Appendix B: Proof of equilibrium to the two state model

Note that we from (8) and (11) have

$$\frac{m(1,\theta_B)}{(1-u_B)} = p_{BB}$$
$$\frac{m(1,\theta_G)}{(1-u_G)} = p_{GG}$$

and hence

$$\frac{(1-u_G)}{(1-u_B)}\frac{m(1,\theta_B)}{m(1,\theta_G)} = \frac{p_{BB}}{p_{GG}} \tag{19}$$

Since $\frac{p_{BB}}{p_{GG}} > 1$ if follows that a sufficient condition that $u_B > u_G$ is $\frac{m(1,\theta_B)}{m(1,\theta_G)} < 1$ or $\theta_B < \theta_G$.

From the value functions for a filled job (5) and (6) we have by use of $J_G^V = J_B^V = 0$ that

$$\rho J_B^E = y - w + \pi p_{BB}(-J_B^E) + (1-\pi)\left[p_{GB}(-J_B^E) + (1-p_{GB})(\frac{q_B}{q_G} - 1)J_B^E\right] \tag{20}$$
$$\rho J_G^E = y - w + \pi p_{GG}(-J_G^E) + (1-\pi)\left[p_{BG}(-J_G^E) + (1-p_{BG})(\frac{q_G}{q_B} - 1)J_G^E\right] \tag{21}$$

Hence

$$\left[\rho + \pi p_{BB} + (1-\pi)\left[p_{GB} + (1-p_{GB})(1-\frac{q_B}{q_G})\right]\right] J_B^E = y - w$$
$$\left[\rho + \pi p_{GG} + (1-\pi)\left[p_{BG} + (1-p_{BG})(1-\frac{q_G}{q_B})\right]\right] J_G^E = y - w$$

and

$$\frac{\left[\rho + \pi p_{BB} + (1-\pi)\left[1 - (1-p_{GB})\frac{q_B}{q_G}\right]\right]}{\left[\rho + \pi p_{GG} + (1-\pi)\left[1 - (1-p_{BG})\frac{q_G}{q_B})\right]\right]} = \frac{J_G^E}{J_B^E} = \frac{q_B}{q_G} \tag{22}$$

where the last equality follows from (7).

Using (12) and (12) we have

$$1 - p_{BG} = \frac{(1-u_B)}{(1-u_G)}(1-p_{BB})$$
$$1 - p_{GB} = \frac{(1-u_G)}{(1-u_B)}(1-p_{GG})$$

Implying that (22) can be written

$$\frac{\left[\rho + \pi p_{BB} + (1-\pi)\left[1 - \frac{(1-u_G)}{(1-u_B)}(1-p_{GG})\frac{q_B}{q_G}\right]\right]}{\left[\rho + \pi p_{GG} + (1-\pi)\left[1 - \frac{(1-u_B)}{(1-u_G)}(1-p_{BB})\frac{q_G}{q_B})\right]\right]} = \frac{q_B}{q_G}$$

and using (19) we get

$$\frac{\left[\rho + \pi p_{BB} + (1-\pi)\left[1 - \frac{p_{BB}}{p_{GG}}\frac{m(1,\theta_G)}{m(1,\theta_B)}(1-p_{GG})\frac{q_B}{q_G}\right]\right]}{\left[\rho + \pi p_{GG} + (1-\pi)\left[1 - \frac{p_{GG}}{p_{BB}}\frac{m(1,\theta_B)}{m(1,\theta_G)}(1-p_{BB})\frac{q_G}{q_B})\right]\right]} = \frac{q_B}{q_G} \tag{23}$$

We have that

$$\frac{q_B}{q_G} = \frac{m(\theta_B^{-1},1)}{m(\theta_G^{-1},1)} = \frac{\theta_B^{-\alpha}}{\theta_G^{-\alpha}} = \left[\frac{\theta_G}{\theta_B}\right]^\alpha$$

and

$$\frac{q_B}{q_G}\frac{m(1,\theta_G)}{m(1,\theta_B)} = \frac{m(\theta_B^{-1},1)}{m(\theta_G^{-1},1)}\frac{m(1,\theta_G)}{m(1,\theta_B)} = \frac{\theta_B^{-\alpha}}{\theta_G^{-\alpha}}\frac{\theta_G^{1-\alpha}}{\theta_B^{1-\alpha}} = \frac{\theta_G}{\theta_B}$$

Condition (23) can now be written

$$\frac{\left[\rho + \pi p_{BB} + (1-\pi)\left[1 - \frac{p_{BB}}{p_{GG}}(1-p_{GG})\frac{\theta_G}{\theta_B}\right]\right]}{\left[\rho + \pi p_{GG} + (1-\pi)\left[1 - \frac{p_{GG}}{p_{BB}}(1-p_{BB})\frac{\theta_B}{\theta_G})\right]\right]} = \left[\frac{\theta_G}{\theta_B}\right]^\alpha \tag{24}$$

It is seen that the LHS of (24) is decreasing in $\frac{\theta_G}{\theta_B}$ and the RHS increasing in $\frac{\theta_G}{\theta_B}$. It follows that there is a unique solution to $\frac{\theta_G}{\theta_B}$ from which all other variables can be found. To proof that $\frac{\theta_G}{\theta_B} > 1$, observe that for$\frac{\theta_G}{\theta_B} = 1$ we have that the RHS of (24) equals one, whereas the LHS is larger than one, hence it follows that $\frac{\theta_G}{\theta_B} > 1$. Note that this implies $\frac{q_G}{q_B} < 1$ and hence $u_G < u_B$.

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